Quantifying the relationship between wealth distribution and aggregate growth in the Ramsey model

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Abstract

This paper provides an explicit characterization of the relationship between growth and wealth distribution among agents when an economy à la Ramsey approaches its steady-state. The result is used to evaluate the distributional consequence of an increase in the income tax rate. © 2002 Published by Elsevier Science B.V.

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1. Introduction

As emphasized by Chatterjee (1994) or Caselli and Ventura (2000), the Ramsey saving behavior implies that the distribution of household financial wealth changes as the aggregate economy undergoes transitional growth. But just how strong is the impact of transitional growth on wealth inequalities? By linearizing the economy around its steady state, we provide an explicit quantification of this influence in function of the deep parameters. We use this characterization to conduct a simple numerical evaluation of the consequence of a fiscal shock on the wealth distribution in the US economy.

2. Convergence and growth in the Ramsey model

All households in the economy are Ramsey savers equipped with an identical intertemporal

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homothetic utility function. The time-invariant relative labor productivity of household \( i \) is given by \( \pi(i) \in [0, \pi_{\text{max}}] \). The aggregate dynamics is driven by the differential system:

\[
\dot{a} = f(a) - c - (n + \delta + x)a \\
\dot{c} = [\theta^{-1}(f'(a) - \delta - \rho) - x]c, \tag{1}
\]

with the appropriate boundary conditions. \( c \) and \( a \) are the levels of consumption and capital stock measured in per capita intensive terms. \( n, x, \delta, \theta \) and \( \rho \) are respectively the rate of population growth, the rate of technological progress, the depreciation rate, the inverse of the elasticity of intertemporal substitution and the discount rate. \( f(.) \) is a neoclassical production function, and \( f'(.) \) its derivative. Noting the share of capital income as \( \alpha \) and the elasticity of substitution associated with \( f(.) \) as \( \epsilon \), a well known result (Barro and Sala-i-Martin, 1995, p. 88) is that any variable in the economy tends towards a balanced growth path (BGP) at speed:

\[
\beta = \frac{1}{2} \left[ - (r^* - (n + x)) + \sqrt{(r^* - (n + x))^2 + \frac{4}{\epsilon \theta} (r^* + \delta)(1 - \alpha) c^* a^*} \right], \tag{2}
\]

where a star denotes variables on the BGP. \( r^* = \rho + \theta x \) is the (net) rate of return.

Following the homogeneity of the utility function, any household \( i \) finds it optimal to have its lifetime wealth \( k(i, t) = a(i, t) + \pi(i)\dot{w}(t) \) grow at a common rate solely determined by aggregate dynamics. \( a(i, t) \) is the level of \( i \)'s financial wealth, while \( \pi(i) \times \dot{w}(t) \), the 'human' wealth, is the discounted sum of her future wages. This can be written as:

\[
\frac{k(t')}{k(t)} = \frac{a(t') + \pi(i)\dot{w}(t')}{a(t) + \pi(i)\dot{w}(t)}, \forall i. \tag{3}
\]

This in turn yields a simple formula describing the distribution of financial wealth dynamics:

\[
a_R(i, t') = \lambda(t, t') a_R(i, t) + [1 - \lambda(t, t')] \pi(i), \forall i. \tag{4}
\]

where \( a_R(i, t) = a(i, t)/a(t) \) is the relative level of financial wealth of household \( i \) and

\[
\lambda(t, t') = \frac{k(t')/k(t)}{a(t')/a(t)} \tag{5}
\]

is a measure of how the share of physical capital in total wealth changes over time. As stressed by Caselli and Ventura (2000), the coefficient \( \lambda(.) \) provides a measure of the influence of transitional aggregate growth on the distribution of financial wealth among households. If \( \lambda(t, t') < 1 \), inequalities are reduced — cross-section convergence (conditional on labor productivity). If \( \lambda(t, t') > 1 \), inequalities increase — divergence. Quantifying \( \lambda(t, t') \) in function of the deep parameters would therefore provide useful information on how the wealth distribution relates to transitional deviations from the BGP.
3. Characterization around the BGP

The following proposition ‘opens up’ the relationship between transitional growth and the distribution of financial wealth by studying the coefficient $\lambda(\cdot)$ around the BGP.

Proposition 1. Assuming that the current capital stock $a(t)$ is not too far off $a^*$:

$$\lim_{t' \to \infty} \lambda(t, t') = \lambda(t, \infty) = 1 - \eta^* \left( \frac{a^* - a(t)}{a(t)} \right),$$

with:

$$\eta^* = 1 - \frac{a^*}{c^*}[r^* - (n + x) + \theta \beta].$$

Proof. Let $\lambda(a, a')$ be the value taken by $\lambda(\cdot)$ when the capital stock changes from $a$ to $a'$ and $k(a)$ the function linking the aggregate lifetime wealth and the capital stock $a$. Eq. (5) implies: $\lambda(a, a') = (k(a')/k(a))/(a'/a)$. It is obvious that $\lambda(a^*, a^*) = 1$. Taking a first order Taylor expansion of $\lambda(a, a^*)$ around $a^*$ yields: $\lambda(a, a^*) = 1 - \eta^*(a^* - a)/a$, where $\eta^*$ is the logarithmic derivative of $\lambda(a, a^*)$ in $a = a^*$, that is, $\eta^* = 1 - (a^*/k^*)(k'(a^*))$, with $k'(a^*) = (dk/da)_{a=a^*}$.

In the vicinity of the BGP, any variable $z$ is driven by the differential equation: $\dot{z} = -\beta(z - z^*)$, with $\beta$ given by (2), and $z^*$ the steady-state value of $z$. This implies that $\dot{z}(z^*) = -\beta$, where $\dot{z}(z^*) \equiv (dz/dz)_{z=z^*}$. Differentiating the system (1) around the BGP and using the latter expression yields:

$$\dot{a}(a^*) = -\beta = (r^* - n - x) - c'(a^*),$$
$$\dot{c}(c^*) = -\beta = c^* \theta^{-1} f''(a^*) \times a'(c^*)$$

From this we have $c'(a^*) = r^* - n - x + \beta$ and $1/a'(c^*) = c'(a^*) = -\beta^{-1} \theta^{-1} c^* f''(a^*)$. Differentiating the law of motion of total wealth ($\dot{k} = (f'(a) - \delta - n - x)k - c$) around the BGP leads to:

$$\dot{k}(k^*) = -\beta = r^* - n - x + k^* f''(a^*) a'(a^*) - c'(a^*) a'(k^*),$$

so that: $k'(a^*) = 1/a'(k^*) = (c'(a^*) - k^* f''(a^*))/(r^* - n - x + \beta)$. Using the expressions of $c'(a^*)$ and the relationship $c^*/k^* = r^* - n - x + \beta$, one gets: $k'(a^*) = 1 + \beta \theta (r^* - n + x)$. Since $k = a + \dot{w}$, the second term on the r.h.s. of this equation indicates how human wealth $\dot{w}$ responds to variations of capital around the BGP. By combining these results, we obtain the expression for $\eta^*$ in the proposition. \(\square\)

The expression in (7) explicitly shows how deep parameters influence the relationship between transitional growth (measured by $(a^* - a(t))/a(t)$ in (6)) and the distribution of wealth. A negative (positive) $\eta^*$ means that growth is associated with an increase (reduction) in inequality.

As illustrated by the proof above, $\eta^*$ ultimately depends on how human wealth responds to capital accumulation: an elasticity of $\dot{w}$ with respect to $a$ which is higher than unity implies a negative $\eta^*$. When this elasticity is strong, the poor — for whom human wealth carries a heavy weight — experience a strong increase in their total wealth. Due to consumption smoothing, they therefore tend
to raise their level of consumption during transition, that is, they tend to reduce their financial wealth. This implies a rise in inequalities in financial wealth.

As noted by Caselli and Ventura (2000), the two key parameters in this mechanism are the elasticity of substitution between factors $\epsilon$, which governs the impact of capital accumulation on wages, and the intertemporal elasticity of substitution $1/\theta$, which determines the preference for consumption smoothing. Quite intuitively, the model shows that more flexibility in the economy (high $\epsilon$ and $1/\theta$) favors a positive link between transitional growth and convergence in financial wealth.

What is surprising in the expression of $\eta$ (7) is that these effects boil down to the term $\beta \times \theta$. $\theta$ enters this term both directly and negatively through its influence on $\beta$. The net influence is positive (see Eq. (2)), as suggested above. The role of $\epsilon$ in $\beta$ can also be seen through Eq. (2).

4. A simple numerical application

Using a standard calibration of the model, we conduct a simple numerical experiment in the vicinity of the BGP. We will specifically evaluate the influence of $\theta$, a key parameter of the convergence behavior.

We consider an economy equipped with a Cobb–Douglas production function ($\epsilon = 1$). The parameters are set as in Barro and Sala-i-Martin (1995), Chapter 2: $n = 0.01$, $x = 0.02$, $\rho = 0.02$, $\delta = 0.05$, $\alpha = 0.75$. A high $\alpha$ is required for the model to accord with observed speeds of aggregate convergence ($\beta$). The economy is on a BGP at time zero. We assume that the financial wealth is distributed among households according to a lognormal law. The mean of the distribution of (relative) financial wealth is unity, while the variance of the corresponding normal distribution is set to 1.77, which produces an initial Gini coefficient of 0.78. According to the 1992 Survey of Consumer Finances, this is the observed Gini coefficient for wealth in the United States (see Diaz Giménez et al., 1997).

Our experiment consists of a small exogenous change in the income tax. It is assumed that the corresponding government revenues are entirely lost. As reported in Krusell and Rios-Rull (1999), the observed average rate of income tax for the United States is 30%. We introduce a one-point increase in the income tax rate, from 30% to 31%. With this shift, the economy will now be 5.9% above its new BGP. This will give rise to a phase of (negative) transitional growth, with the associated effect on the wealth distribution. The results are shown in Table 1.

The positive $\eta$'s indicate that there is cross-section convergence whenever the economy grows. With the economy converging from above to a new BGP, the effect on wealth distribution will be divergence: the $\lambda$'s are greater than unity. These simulations therefore illustrate a paradoxical consequence of the Ramsey model: because it leads to a lower BGP an increase in the income tax

<table>
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<tr>
<th>$\theta$</th>
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<th>$\gamma$</th>
<th>$\lambda$</th>
<th>Gini (final)</th>
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rate, with realistic parameter settings, will result in a rise of inequalities. The conventional wisdom that there may exist a trade-off between growth and equality thus fails in the Ramsey model with calibrated parameters. Here, on the contrary, negative (positive) growth is associated with an increase (reduction) in inequalities, following the economic logic given above: as the human wealth component of total wealth declines slowly during the transition, relative dissaving is stronger for the poor, implying divergence.

But how strong is this effect? The Gini coefficients at the new BGP provide a measure of how much a one-point increase in the income tax rate affects the distribution of wealth. The main result is that the distributional impact of the fiscal shock is fairly modest for calibrated values of $\theta$, in the range of $\theta = 3$.

For low values of this parameter, the distributional impact is, on the contrary, significant, resulting in a two-point shift of the Gini coefficient for $\theta = 0.5$.

5. Conclusion

We have provided an explicit characterization of the relationship between growth and wealth distribution in the neighborhood of the steady state. As the Ramsey model does not admit any closed-form solutions, this is useful to study the influence of transitional growth on the reduction, or increase, of inequalities. Simple numerical applications suggest that for realistic values of the parameters, particularly the intertemporal elasticity of substitution, the convergence effect to be expected from a Ramsey savings behavior is rather modest. These calculations also illustrate one paradoxical implication of optimal savings: still with realistic values of parameters, an increase in the income tax leads to a rise of inequalities in financial wealth, albeit small.

Finally, it is important to note that our algebraic results can be extended to the income distribution without difficulty.

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References